

About matter and dark-energy domination eras in R^n gravity or lack thereof

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We provide further numerical evidence which shows that R^n models in $f(R)$ metric gravity whether produces a late time acceleration in the Universe or a matter domination era (usually a transient one) but not both. Our results confirm the findings of Amendola *et al.* [1–3], but using a different approach that avoids the mapping to scalar-tensor theories of gravity, and therefore, dispense us from any discussion or debate about frames (Einstein *vs* Jordan) which are endemic in this subject. This class of models has been used extensively in the literature as an alternative to the dark energy, but should be considered ruled out for being inconsistent with observations. Finally, we discuss a caveat in the analysis by Faraoni [4], which was used to further constrain these models by using a chameleon mechanism.

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I. INTRODUCTION

$f(R)$ theories of gravity are perhaps the most straightforward modification of general relativity (GR), providing an extra geometric component which in some particular cases is capable of generating the accelerated expansion of the Universe manifested in supernovae Ia [5]. A large amount of literature has been accumulated in the past ten years about this kind of alternative theories of gravity and is beyond the scope of the present article to make justice to this vast subject (see Refs. [6] for a thorough review). Although some specific $f(R)$ models have shown to be consistent with certain astronomical observations, within the Solar System and also cosmological, not every model has the same success, for instance the $f(R) = \lambda R_n (R/R_n)^n$ model, simply referred in the literature as to R^n . Recently, Amendola *et al.* [1, 2] performed a detailed analysis on the cosmological viability of several classes of $f(R)$ models, including R^n . Using a dynamical system approach, they concluded that for this latter the usual matter era that precedes the accelerated phase with a scale factor $a(t) \sim t^{2/3}$ is generically replaced by a non standard era with $a(t) \sim t^{1/2}$ (c.f. Ref. [7] for a complementary analysis), and in the cases where it is possible to achieve a usual matter domination epoch the accelerated expansion is not possible. In any instance, the conclusion was that such a model is simply unable to reproduce the observed features of our Universe without the addition of some form of dark energy.

These results have been, however, the object of a debate concerning two issues: 1) the *frames* (Einstein *vs* Jordan) used in the scalar-tensor (ST) approach to ana-

lyze the R^n and other models [3, 8, 9]; and 2) the analysis of the phase space [10, 11].

Since the R^n model has been and keeps being considered in the literature (see a complete list of references in [4]) it is important to settle this question with an independent method and beyond any reasonable doubt.

In this brief report we reanalyze the cosmological case of the R^n model using a different method and spanning a wide range of n . Our technique does not involve what is usually called the *scalar-tensor approach* (ST) where a scalar field $\phi = f_R$ is defined in order to map $f(R)$ theories to a Brans–Dicke like theory with $\omega = 0$ and a potential. Instead, we promote the Ricci scalar itself as an independent degree of freedom [12, 13] and in this way we circumvent the potential drawbacks associated with the ST approach (e.g. multivalued scalar-field potentials), and in addition avoid the long standing issue about frames (Jordan *vs* Einstein) which plagues not only the ST method, but also the analysis of scalar-tensor theories themselves, and which gave rise precisely to the unnecessary debate mentioned above about the cosmological viability of R^n and other class of $f(R)$ theories. As we will show, our approach leads to a rather “friendly” system of equations which are much more simple to treat than other systems found in the literature and that can be easily solved numerically. We had used this method before in the analysis of compact objects [12] and more recently in cosmology using different $f(R)$ models [13, 14]. For the cosmological analysis at hand, we shall consider the same tools developed in [13] and adapt them to the case R^n .

Our analysis supports the general conclusions of [1, 2] and [3] (although we do not commit ourselves in assessing the soundness of their phase-space analysis) providing a second, independent, strong and unambiguous piece of evidence showing that the specific R^n model is not cosmologically viable. In the next section we discuss in

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detail our findings that lead to such conclusion, and we also argue that the analysis put forward by Faraoni [4] to constrain these kind of model in the light of the Solar System tests using a chameleon mechanism, is ill founded and requires a deeper review.

II. $f(R)$ THEORIES

The action in $f(R)$ gravity is given by:

$$S[g_{ab}, \psi] = \int \frac{f(R)}{2\kappa} \sqrt{-g} d^4x + S_{\text{matt}}[g_{ab}, \psi], \quad (1)$$

where $\kappa \equiv 8\pi G_0$ (we use units where $c = 1$), $f(R)$ is a sufficiently differentiable but otherwise *a priori* arbitrary function of the Ricci scalar R and ψ represents schematically the matter fields. The field equation obtained from Eq. (1) is:

$$f_R R_{ab} - \frac{1}{2} f g_{ab} - (\nabla_a \nabla_b - g_{ab} \square) f_R = \kappa T_{ab}, \quad (2)$$

where f_R indicates $\partial_R f$, $\square = g^{ab} \nabla_a \nabla_b$ is the covariant D'Alembertian and T_{ab} is the energy-momentum tensor of matter associated with the ψ fields. From Eq. (2) it is straightforward to obtain the following equation and its trace [12, 13]

$$G_{ab} = \frac{1}{f_R} \left[f_{RR} \nabla_a \nabla_b R + f_{RRR} (\nabla_a R) (\nabla_b R) - \frac{g_{ab}}{6} (R f_R + f + 2\kappa T) + \kappa T_{ab} \right], \quad (3)$$

$$\square R = \frac{1}{3f_{RR}} \left[\kappa T - 3f_{RRR} (\nabla R)^2 + 2f - R f_R \right], \quad (4)$$

where $(\nabla R)^2 := g^{ab} (\nabla_a R) (\nabla_b R)$ and $T := T^a_a$.¹ Equations (3) and (4) are the basic equations we use in order to find the cosmic evolution in the model R^n . We have employed this system of equations in the past for several applications, and the reader is invited to consult Refs. [12, 13] for a detail discussion of this approach. Before analyzing the cosmological situation, it is important to make some remarks regarding several issues that arise in this particular model but not in other viable $f(R)$ models. In $f(R)$ theories, one usually demands the conditions $f_R > 0$ and $f_{RR} > 0$. The first one is imposed in order to have a positive definite effective gravitational constant $G_{\text{eff}} := G_0/f_R$, while the second condition, is considered in order to avoid instabilities around a possible de Sitter background [15]. We would like to elaborate more about this second point.

In [12, 13] we introduced the potential $V(R)$ such that $V_R := (2f - R f_R)/3$ which was relevant for tracking the possible de Sitter points allowed by the theory and which correspond to trivial solutions of Eq. (4) in vacuum. These trivial solutions are given by $R = R_1 = \text{const.}$ such that $V_R(R_1) = 0$, assuming $f_{RR}(R_1) \neq 0$, where the effective cosmological constant is $\Lambda_{\text{eff}} = R_1/4$. This explains qualitatively why $f(R)$ theories having a de Sitter point can potentially produce an accelerated expansion when $R \rightarrow R_1$ and $\rho_{\text{matt}} \rightarrow 0$ as the Universe evolves. Now, if $f_{RR}(R_1) = 0$, one should define instead $\tilde{V}_R = (2f - R f_R)/(3f_{RR})$, being that f_{RR} appears in the denominator of Eq. (4). Since this situation happens generically in the model R^n , we shall consider \tilde{V}_R and not V_R . Related with the stability analysis is the mass of scalar mode around a de Sitter point $R = R_1$: $\tilde{m}^2 := \tilde{V}_{RR}(R_1) = (m^2 - f_{RRR} \tilde{V}_R / f_{RR})_{R_1}$, where $m^2 = [f_R - R f_{RR}]_{R_1} / [3f_{RR}(R_1)]$, and if $f_{RR}(R_1) \neq 0$ then $\tilde{m}^2 \equiv m^2$ since $\tilde{V}_R(R_1) = 0$. Usually when $f_{RR}(R_1) \neq 0$, m^2 is negative if $f_{RR}(R_1) < 0$ (assuming $f_R(R_1) > 0$ and $R_1 > 0$), and in that case instabilities may develop rapidly in time [15]. Thus one should consider theories where $f_{RR}(R_1) > 0$ ², and in this case $m^2 > 0$ if the critical point at R_1 is a minimum of $\tilde{V}(R)$ or $V(R)$.

Let us now focus on $f(R) = \lambda R_n (R/R_n)^n$, where λ is a dimensionless constant, and R_n is another constant which in general depends on the choice of n , and settles the built-in scale. In practice $R_n = \alpha_n H_0^2$, where α_n is a dimensionless constant and H_0 is the current Hubble parameter. One then has $f_{RR} = \lambda n(n-1)(R/R_n)^{n-2} R_n^{-1}$. We shall not consider the case $n = 0$ nor 1 because $n = 1$ corresponds to general relativity (GR), for which some sort of dark energy or cosmological constant is required in order to explain the accelerated expansion, and for $n = 0$ the theory “disappears” (i.e. it is too simple), so from now on we assume $n \neq 0, 1$. The condition $f_{RR} > 0$ holds in general provided $n > 1$ or $n < -1$, assuming in both cases $R > 0$, and f_{RR} may vanish only at $R = 0$ (we call this point R_0) or when $R \rightarrow \infty$ (R_∞). We shall not consider $n < 0$ because then $f_R = \lambda n(R/R_n)^{n-1}$ becomes negative (assuming $R > 0$), and the condition $G_{\text{eff}} > 0$ is violated. The quantity f_R also vanishes at R_0 or R_∞ , depending on n . Finally, $\tilde{V}_R = R^2(2-n)/[3n(n-1)]$, and thus $\tilde{V}(R) = R^3(2-n)/[9n(n-1)] + \text{const.}$ ³ For $n = 2$,

¹ We assume in all the article that a subscript R stands for $\partial/\partial R$.

² The model $f(R) = R - \mu^4/R$ has a de Sitter point at $R_1 = \mu^2\sqrt{3}$ and the mass is negative: $m^2 = -\mu^2\sqrt{3}$.

³ Had we considered the potential $V(R)$ instead of $\tilde{V}(R)$ one would obtain $V_R(R) = 0 = \lambda R_n(2-n)(R/R_n)^n/3$, which for $n \neq 2$ and positive has $R = 0$ as the only stationary solution in vacuum. Therefore in practice $V(R)$ and $\tilde{V}(R)$ single out the same location for the extrema R_0 and R_1 which correspond to the stationary (trivial) vacuum solutions of Eq. (4) alluded in the main text for the R^n model.

$\tilde{V}(R) = \text{const.}$, and any $R = R_1 \neq 0$ can be a de Sitter point, the specific value R_1 depends on the initial conditions when integrating the equations. Apart from this “degenerate” case, \tilde{V}_R vanishes only at R_0 . Therefore, for $n \neq 2$ the model R^n does not admit de Sitter points and would only be able to generate an accelerated era in a rather transient fashion since far in the future the matter contribution dilutes and if R reaches some equilibrium point it will only be at R_0 which corresponds to $\Lambda_{\text{eff}} = 0$. The mass $\tilde{m}^2 = \tilde{V}_{RR}(R) = 2R(2-n)/[3n(n-1)]$, which in this case is to be evaluated at $R = R_1$ or $R = R_0$ (i.e. $R = 0$) vanishes identically for $n = 2$, regardless of the value of the de Sitter point R_1 . Notice that $\tilde{m}^2 = 2m^2/n$, where m was defined above. On the other hand, for $n \neq 2$ the only critical point of $\tilde{V}(R) \sim R^3 + \text{const.}$ is a saddle point at $R = 0$ (R_0), where, as mentioned before $\tilde{V}_R(R_0)$ vanishes, and where \tilde{m}^2 vanishes as well regardless of the value of n (we assumed $n \neq 0, 1$).

When a de Sitter point $R_1 \neq 0$ exists in vacuum $R_1 = 4\Lambda_{\text{eff}} = 12H_{\text{vac}}^2 \neq 0$ (c.f. Eq. (7) with $\rho_X = \Lambda_{\text{eff}}/\kappa$ and in the limit $\rho \rightarrow 0$). However, with $R_0 = 0$, one is led to $\Lambda_{\text{eff}} = 0 = H_{\text{vac}}^2$. Faraoni [4] overlooked this fact and obtained instead $m^2 = \frac{1}{3}(f_R/f_{RR} - R)_{R_0} = (2-n)R_0/[3(n-1)]$,⁴ assuming $R_0 \neq 0$, and thus concluding $m^2 \neq 0$ for $n \neq 2$. As we just argued, this conclusion is incorrect since the only “de Sitter” point in the R^n model is $R_0 = 0$, for $n \neq 2$ and therefore $m^2 \equiv 0$.⁵ The analysis in [4] relies on the fact that $m^2 \neq 0$ and requires the latter to be sufficiently large for the chameleon mechanism to ensue, in which case the author concluded $n = 1 + \delta$ with $0 \leq \delta \leq 5 \times 10^{-30}$. Again, that analysis would be valid if the model had a true de Sitter point at $R_1 \neq 0$ for $n \neq 2$. In light of the previous discussion, we see that the analysis in [4] is no longer sustained nor even required since no matter the value of n (with $n \neq 0, 1$) the scalar mode is massless. In reality, the chameleon requires a “thin shell” condition and an effective mass [16], both depending on the density of the environment, so m^2 by its own does not suffice to analyze such mechanism. But, if it were the case, then the R^n model would be discarded

⁴ Notice the missing factors of ‘2 and n with respect to \tilde{m}^2 . The difference arises because in our definition of \tilde{m}^2 we did not assume anything about the critical point precisely because f_{RR} might vanish there. Nonetheless, such factors are irrelevant for $R_0 = 0$ since then $m^2 \equiv 0 \equiv \tilde{m}^2$.

⁵ In [4] the range of the scalar mode was denoted by $s(n)$ which is given by $s(n) = 1/m \sim 1/\tilde{m}$, but since both m and \tilde{m} are zero at R_0 and at R_1 for any $n \neq 0, 1$ then $s(n) \rightarrow \infty$, contrary to what was found in [4] for $n \neq 2$, where it was assumed that $H_{\text{vac}}^2 = R_0/12 \neq 0$, denoted by H_0^2 in that reference. Here H_0 is the actual cosmological constant where all forms of matter (ordinary and the “geometric dark energy”) are taken into account, while H_{vac} is the Hubble expansion when the ordinary matter is neglected and when it is evaluated at the stationary solution of Eq. (6). So in [4] no distinction is made between H_0 and H_{vac} .

automatically even if δ were within the above interval (with $\delta \neq 0$) since, the scalar mode being massless, one of the Post-Newtonian parameter would be $\gamma \sim 1/2$ whose relative difference with $\gamma_{\text{GR}} = 1$ is more than four orders of magnitude larger than the maximum value admitted by observations $|\gamma - 1| \lesssim 2.3 \times 10^{-5}$ [17].⁶

In the next section we perform a numerical analysis of the full cosmological equations and show that within the model R^n , including the case $n = 2$, an adequate matter dominated era followed by a satisfactory accelerated expansion is very unlikely or impossible to happen.

III. COSMOLOGY IN $f(R)$

We assume a homogeneous and isotropic space-time described by the Friedmann-Robertson-Walker metric:

$$ds^2 = -dt^2 + a^2(t) [dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)] \quad , \quad (5)$$

where we have taken the flat case $k = 0$. From Eqs. (3) and (4) we have,

$$\ddot{R} = -3H\dot{R} - \frac{1}{3f_{RR}} [3f_{RRR}\dot{R}^2 + 2f - f_R R + \kappa T] \quad , \quad (6)$$

$$H^2 = \frac{\kappa}{3} (\rho + \rho_X) \quad , \quad (7)$$

$$\dot{H} = -H^2 - \frac{\kappa}{6} \left\{ \rho + \rho_X + 3(p_{\text{rad}} + p_X) \right\} \quad . \quad (8)$$

where $\dot{} = d/dt$ and $H = \dot{a}/a$, is the Hubble expansion. In the above equations we have included the energy density ρ associated with matter (baryons and dark matter) and radiation, as well as the geometric dark energy density ρ_X and pressure p_X given explicitly by

$$\rho_X = \frac{1}{\kappa f_R} \left\{ \frac{1}{2} (f_R R - f) - 3f_{RR} H \dot{R} + \kappa \rho (1 - f_R) \right\} \quad , \quad (9)$$

$$p_X = -\frac{1}{3\kappa f_R} \left\{ \frac{1}{2} (f_R R + f) + 3f_{RR} H \dot{R} - \kappa (\rho - 3p_{\text{rad}} f_R) \right\} \quad . \quad (10)$$

These quantities can also be obtained from a covariant and conserved energy-momentum tensor associated with the geometric modifications to GR [13].

Notice that the expression for the Ricci scalar computed directly from the metric (5) is given by $R =$

⁶ It is important to stress that the “weak-field”, linear or Newtonian limits in $f(R)$ theories are usually studied around a maximum or minimum of $\tilde{V}(R)$. The fact that in this case the critical point is a saddle point indicates that a full non linear analysis is required around that point and that such limits are to be reconsidered in R^n gravity. Notice that Eq. (4) reads explicitly $\square R = \frac{\kappa T R_n (R/R_n)^{2-n} + \lambda(2-n)R^2}{3\lambda n(n-1)} + (2-n)(\nabla R)^2/R$.

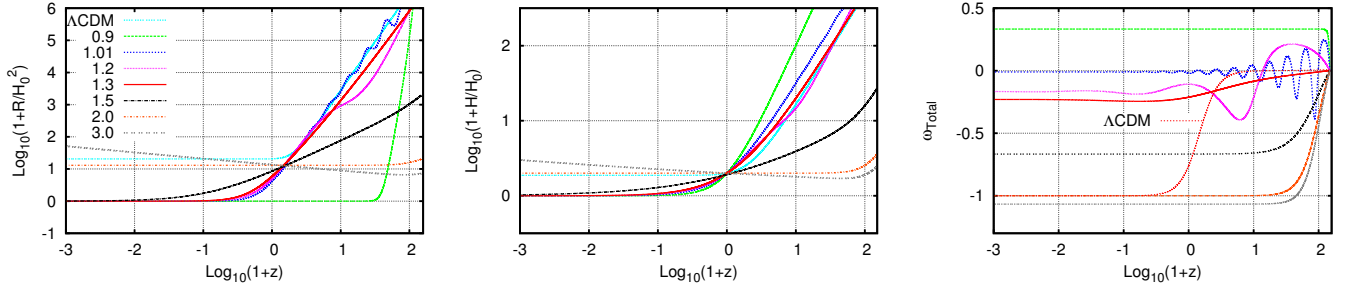


FIG. 1. (color online) Ricci scalar (left panel), Hubble parameter (middle panel) and the total EOS ω_{tot} in R^n gravity for several values of the exponent n , taking $\lambda = 1$ and the constants $\alpha_n = R_n/H_0^2$ as follows: $\alpha_{0.9} \sim 577.85$, $\alpha_{1.01} \sim 404.84$, $\alpha_{1.2} \sim 2.02$, $\alpha_{1.3} \sim 1.07$, $\alpha_{1.5} \sim 8 \times 10^{-4}$, $\alpha_2 \sim 7.9 \times 10^{-6}$, $\alpha_3 \sim 2.6 \times 10^{-6}$. The Λ CDM model is plotted for reference. The plots of the middle and right panels correspond to the cases of the left panel. For $n < 2$ the Ricci scalar and the Hubble expansion approach zero as $z \rightarrow -1$, while these quantities keep growing for $n > 2$. The model $n = 2$ has an effective cosmological constant and produces $\omega_{\text{tot}} = -1$ in the far future. However, it does not possess a sufficiently large matter dominated epoch with $\omega_{\text{tot}} \sim 0$, as one can appreciate from the right panel. None of the models decelerate and accelerate as the Λ CDM model.

$6(\dot{H} + 2H^2)$ which is, as one can check, compatible with the previous evolution equations. Therefore, one can use this latter instead of Eq. (8). The modified Friedmann Eq. (7) is used only to check the consistency and accuracy of our numerical code at every time step and also to fix the initial data (see Ref. [13] for the details). We shall not use t as independent variable but $\alpha = \ln(a/a_0)$, where a_0 is the present value of a . The corresponding differential equations can be found in [13].

The matter variables obey the conservation equation $\dot{\rho}_i = -3H(\rho_i + p_i)$ for each fluid component labeled by i (with $p_{\text{bar,DM}} = 0$ and $p_{\text{rad}} = \rho_{\text{rad}}/3$) which integrates straightforwardly and gives rise to the usual expression for the energy density of matter plus radiation: $\rho = (\rho_{\text{bar}}^0 + \rho_{\text{DM}}^0)(a/a_0)^{-3} + \rho_{\text{rad}}^0(a/a_0)^{-4}$, where the knotted quantities indicate their values today. The X -fluid variables (9) and (10) also satisfy a conservation equation similar to the one above, but with an equation of state (EOS) $\omega_X := p_X/\rho_X$ that evolves in cosmic time.

The different domination eras can be tracked via the total EOS defined by $\omega_{\text{tot}} = (p_{\text{rad}} + p_X)/(\rho + \rho_X)$ which using Eqs. (9) and (10) yields

$$\omega_{\text{tot}} = -\frac{1}{3} \left[\frac{\frac{1}{2}(f_R R + f) + 3f_{RR}H\dot{R} - \kappa\rho}{\frac{1}{2}(f_R R - f) - 3f_{RR}H\dot{R} + \kappa\rho} \right]. \quad (11)$$

For instance, during the radiation, matter and geometric-dark-energy dominated eras $\omega_{\text{tot}} \sim 1/3, 0, -1$ respectively. Clearly such values are also correlated with the behavior of the dimensionless densities $\Omega_i = \kappa\rho_i/(3H^2)$ which satisfy the constraint $\Omega_{\text{rad}} + \Omega_{\text{matt}} + \Omega_X = 1$ where $\Omega_{\text{matt}} := \Omega_{\text{bar}} + \Omega_{\text{DM}}$. The capability of the R^n model for reproducing the correct domination eras will be assessed by the behavior of Ω 's and ω_{tot} during the cosmic evolution relative to the Λ CDM model. In this regard it is important to remark that the X -fluid could behave as

a matter, radiation or even as a “ghost” fluid (one with $\rho_X < 0$) depending on the value of the exponent n , and therefore it could lead to an inadequate evolution history of the Universe. We discuss these possibilities in the next section.

IV. NUMERICAL RESULTS AND DISCUSSION

We integrate the differential equations starting at some redshift $z = a_0/a - 1$, say $z \sim 150$, by assuming matter domination for all the n 's in the R^n model that we analyze. We obtain the initial conditions as described in [13] and find that varying them in several ways it turns out impossible to recover the actual abundances of the different components at present time while having an adequate accelerating phase. Here we take $\lambda = 1$ but our conclusions do not change by choosing other (positive) values. This means that compared to the Λ CDM model, the Universe expands faster or slower depending on n but it never reproduces the correct accelerated expansion and matter domination eras within the same model; it reproduces one or the other in the best of scenarios but not both. Figure 1 shows the evolution of the Hubble parameter and the Ricci scalar from the past at $z \sim 150$ to the far future $z \rightarrow -1$ (the current time corresponding to $z = 0$). Notice that for $n = 2$ the model admits a de Sitter solution with $R \rightarrow R_1 \approx 12H_0^2$ as the Universe evolves towards the present time. Since we have taken into account the matter terms, the previous equality does not hold exactly, but approximates very well to the expected value, in agreement with our previous analysis of Sec. II. From Figure 1 (right panel) we appreciate that for this n , the EOS $\omega_{\text{tot}}^{z=0}$ is close to $\omega_{\text{obs}} \sim -0.75$, which is the required value to explain the current accelerated ex-

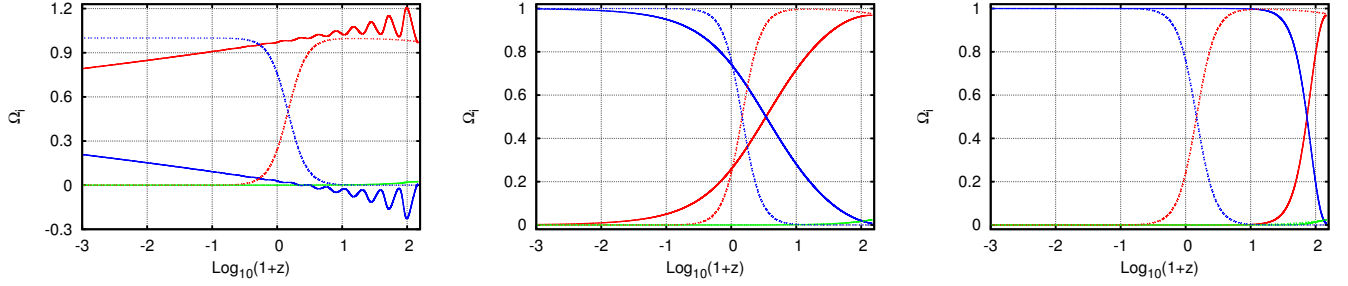


FIG. 2. (color online) Evolution of Ω_{matt} (red solid line), Ω_X (blue solid line) and Ω_{rad} (green solid line) in R^n gravity for $n = 1.01$ (left panel), $n = 1.3$ (middle panel) and $n = 2$ (right panel). For reference the corresponding quantities of the Λ CDM model are included in each panel (dashed lines). Notice from the left panel that Ω_X can be negative and $\Omega_{\text{matt}} > 1$.

pansion of the Universe. Nevertheless, the matter epoch is very short as compared with the Λ CDM model. For any other value of n , a de Sitter point is never reached, instead $R \rightarrow 0$, and $H \rightarrow 0$ for $0 < n < 2$ and R and H grows in the future for $n > 2$ (c.f. equation in footnote 6).

The Λ CDM model compatible with the supernovae data shows that matter starts dominating for $z \gtrsim 0.45$ and dark energy for $z \lesssim 0.45$ which correspond respectively to $\omega_{\text{tot}} \gtrsim -0.5$, and $\omega_{\text{tot}} \lesssim -0.5$ reaching $\omega_{\text{tot}} \gtrsim -10^{-2}$ for $z \gtrsim 5$, and $\omega_{\text{tot}} \lesssim -0.75$ for $z \lesssim 0$. The Universe starts accelerating when $\omega_{\text{tot}} < -1/3$ at $z \sim 0.8$. Figure 1 shows that for $n \sim n_c$ with $n_c \approx 1.285$ a sufficiently large matter dominated era exists with $|\omega_{\text{tot}}| \ll 1$, but approaches the value $\omega_{\text{tot}}^{z=0} \approx -0.212$ which is incompatible with $\omega_{\text{obs}} \sim -0.75$. For $n < n_c$, there is never a matter domination era and $\omega_{\text{tot}}^{z=0}$ is always far from ω_{obs} , and it can even be positive. In particular, for $n < 1$, which we include for illustrative purposes as it violates the condition $f_{RR} > 0$, the model behaves as radiation dominated with $\omega_{\text{tot}} \sim 1/3$ and the derivatives f_R and f_{RR} blow up when $R \rightarrow 0$. Finally, for $n > n_c$, there is never a matter dominated epoch, but just a transient one with $\omega_{\text{tot}} < 0$ interpolating monotonically between $\omega_{\text{tot}} \sim 0$ and a negative value at $z = 0$. Among these values, for $n > 3$ basically all the models behave identically with $\omega_{\text{tot}} \sim -1.067$ as $z \rightarrow -1$. Figure 2 shows the corresponding evolution of the fractions Ω_X , Ω_{matt} and Ω_{rad} for a prototype of examples that qualitatively encompasses the rest of the cases, and are compared with the Λ CDM model. For $n \sim n_c$ the abundances are similar to Λ CDM, particularly at the present epoch ($z = 0$), but as we mentioned above, the model is unable to accelerate the Universe properly. For $n > n_c$ the matter domination epoch is very short (in agreement with the behavior of ω_{tot} in Fig. 1). Finally, for $n < n_c$, Ω_X can even become negative, with Ω_{matt} possessing phases of superdomination (i.e. $\Omega_{\text{matt}} > 1$ in those phases) where ω_{tot} can become positive. In particular when we take $n = 1 + \epsilon$ with

$|\epsilon| \ll 1$, the denominator in Eq. (6), or equivalently in Eq. (4), becomes very small (c.f. equation in footnote 6) producing an important contribution on the r.h.s. of the differential equation for R . The cosmological evolution for such values of n is then rather different from general relativity. For instance, taking $n = 1.01$ we appreciate from Fig. 1 (right panel) that ω_{tot} oscillates around a value near zero due to the oscillations of R (left panel). This oscillatory behavior can also be appreciated in Ω_X and Ω_{matt} from Fig. 2 (left panel). The amplitude of these oscillations are damped so that $\omega_{\text{tot}} \rightarrow 0$ in the present ($z = 0$) and future, thus the Universe does not accelerate. On the other hand, for $\epsilon < 0$, say $\epsilon = -0.1$ ($n = 0.9$), $\omega_{\text{tot}} \sim 1/3$, as mentioned before, and the Universe behaves as radiation dominated.

Carlioni *et al.* [11] following [10], performed a cosmological analysis using a dynamical system approach based on a first order system of equations which is different from ours and which was useful for a qualitative description of the cosmological evolution in R^n gravity. In their approach they found the fixed points (stable, unstable or saddle) of this and other $f(R)$ models which can represent the matter or the accelerated phases in the Universe. As argued by these authors some of these fixed points are different from the ones found in [2] which, as they stressed, might change the conclusions therein. Nevertheless, the authors in [11] acknowledge that their analysis is only qualitative as the fixed points might not even be connected, and that an accurate numerical analysis is required. This is precisely that we have performed here.

In summary, the homogeneous and isotropic cosmology in R^n gravity seems to show a complete disagreement with what is required to explain the current features of the actual Universe. Since our numerical integration was performed by including the whole mixture of components in the Universe, even if radiation is relatively small, and without any identification with a scalar-tensor theory in any frame whatsoever, the generic problems in

the R^n model seem real and are not due to any artifact concerning the ST approach or due to any inconsistency regarding a *phase space* analysis as objected in [8–11]. Thus, we strongly support the conclusion that R^n is not a cosmologically viable candidate, unless a curvature $k \neq 0$ changes things dramatically and makes everything fit with observations. But in such occurrence, a non standard inflationary paradigm has to be called for explaining the origin of cosmological perturbations.

In [13, 14] we explored other $f(R)$ models that can produce a successful background cosmology (i.e. without taking into account perturbations) but needless to say, a

detailed scrutiny is required in all possible ambits before considering $f(R)$ theories as a serious threat to general relativity.

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